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Nonuniversality in short-time critical dynamics

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Abstract. We study the behaviour of dynamical critical exponents of the two-dimensional Ising model with a line of defects. Simulations done at an early time (first 100 Monte Carlo steps) reveal that the critical exponent θ of Janssen *et al* (*Z. Phys.* B **73** 539) depends on the strength of the exchange coupling constant (J') of the altered line. On the other hand, our simulations permit us to conclude that the dynamical critical exponent z is not sensitive to changes in J'. In addition, we investigate the possible invariance of the anomalous dimension (x_0) of the magnetization at the beginning of the process.

Many results have recently been obtained concerning the critical dynamical behaviour of statistical models at an early time. This kind of investigation was motivated by analytical and numerical results contained in the papers of Janssen *et al* [1] and Huse [2] pointing to a new dynamical critical exponent (θ) describing the now-called 'critical initial slip' phenomena. This exponent describes the raising of the magnetization at very short times

$$M(t) \sim m_0 t^{\theta} \tag{1}$$

when a system initially in random states, with a small magnetization, is quenched to the critical temperature and evolves with the dynamics of model A. Simulations done for the Ising and Potts ($q = 3$) model indicate that the new exponent θ does not depend on the kind of dynamics (Metropolis, heat-bath or Glauber). In addition, generalized Binder's cummulant [3] as well as other sample averages [4, 5] which scale as L^0 have proved to be useful in determining the dynamical exponent z which relates time and spatial correlation length ($\tau \propto \xi^z$). The power of this approach was also verified in nonequilibrium models such as the majority vote model [6]. In this paper we investigate the behaviour of the dynamical critical exponents (z and θ) when a marginal operator is present in the Hamiltonian. Although the presence of a marginal operator has well known consequences for the static critical behaviour [7], very little is known about the dynamical one. To the best of our knowledge the only study on this subject was recently conducted by Li *et al* [8] for the Ashkin–Teller model which interpolates between Ising and $q = 4$ Potts model, exhibiting a continuous varying exponent (ν) for the correlation length (ξ). They have also detected a nonuniversal behaviour of the dynamical exponent z. In this paper we study the two-dimensional Ising model with a line of defects (different coupling constants between spins along a given line) to verify the influence of the marginal operator on the dynamical exponents θ and z. In fact, we found that along the altered line the exponent θ depends on the ratio (J'/J). On the contrary, we detected that no changes related to the exponent z, a hypothesis that we used to obtain the dependence of the critical exponent η on the ratio of couplings (J'/J). Finally, we have investigated the behaviour of the anomalous dimension x_0 of the magnetization at the beginning of the process. At least within the precision of our calculations, that value was kept constant despite the value of the coupling constant (J'). In the following we present the model and results of the simulations that we performed.

The two-dimensional isotropic Ising model undergoes a spontaneous symmetry breaking [9] at a critical value of the temperature given by $T = 2J/k\ln 2$. In the neighbourhood of that temperature, the correlation length diverges as $(T - T_c)^{-1}$ which means that $\nu = 1$. In addition, the magnetization vanishes like $(T - T_c)^{1/8}$ which leads to the critical exponent $\beta = \frac{1}{8}$. At long distances the correlation energy–energy behaves, at the critical temperature, as r^{-2x_ε}, where x_ε is the anomalous dimension of the energy satisfying the relation: $x_\varepsilon + 1/v = 2$. Thus, the introduction of a different coupling (J'), along just one line of the lattice $(d = 1)$, fulfils the necessary condition $x_{op} = d = 1$ to obtain nonuniversality [7]: the presence in the Hamiltonian of an operator which scales as r^{-d}. This, in fact, was the result obtained by Bariev [10] and McCoy and Perk [11], later rederived by Peschel and Schotte [12] in the context of quantum chains. After the work of Cardy [13], stressing the importance of the conformal invariance in two-dimensional systems, the Ising model with a line of defects was revisited by Turban [14]. Following this, many-body techniques [15] were used to investigate its quantum analogue [16–18], however, this model still deserves further attention [19]. It is interesting to note that the 'global' exponent ν, as well as the critical temperature, does not change when the value of the coupling (J') is altered. The same is not true for the exponent $(\eta^* = 2\beta^*)$ of the correlation function *along* the line of defects (at $T = T_c$). This 'local' exponent depends continuously on the value of the coupling (J') as

$$\eta^* = \left(\frac{2}{\pi}\tan^{-1} K\right)^2 \tag{2}$$

where

$$K = \tanh(J_1/kT_c)/\tanh(J/kT_c)$$

and J_1 is the dual of the modified coupling J'

$$\exp(-2J_1/kT_c) = \tanh(J'/kT).$$

Our investigation began by following the evolution of the magnetization when samples are sharply prepared with $m_0 \neq 0$ but small. We measured the average over samples (10^5) of the magnetization for two lines (one pure, the other with defects). In addition, we also calculated the magnetization over the entire sample. Simulations were done for lattices with $36 \leqslant L \leqslant 72$ and the update followed from heat-bath dynamics. Because our interest is in the development of magnetization of the lines, we prepared all the samples with the same initial magnetization for each line. Figures 1(a) and (b) present the raise of magnetization for three cases: pure line, line with defects and entire sample, when $J'/J = 2$ and 0.5. Note the similarity between the curves associated to the pure line and entire sample in both cases. Even when working with small lattices we note that the effect of the line of defects restricts itself to its neighbourhood. The conclusion is that the exponent θ depends on the coupling constant of the defect line (see figure 2) which occurs with the static critical index η^*. Dynamical universality (same results as obtained by heat-bath and Metropolis updating) was observed in the simulations found in the pure Ising model [20], although bigger deviations could be seen when $J'/J \neq 1$.

The next step is to check the plausible hypothesis that the dynamical critical exponent z is not changed by the line of defects. As pointed by Li [21], the short-time behaviour of

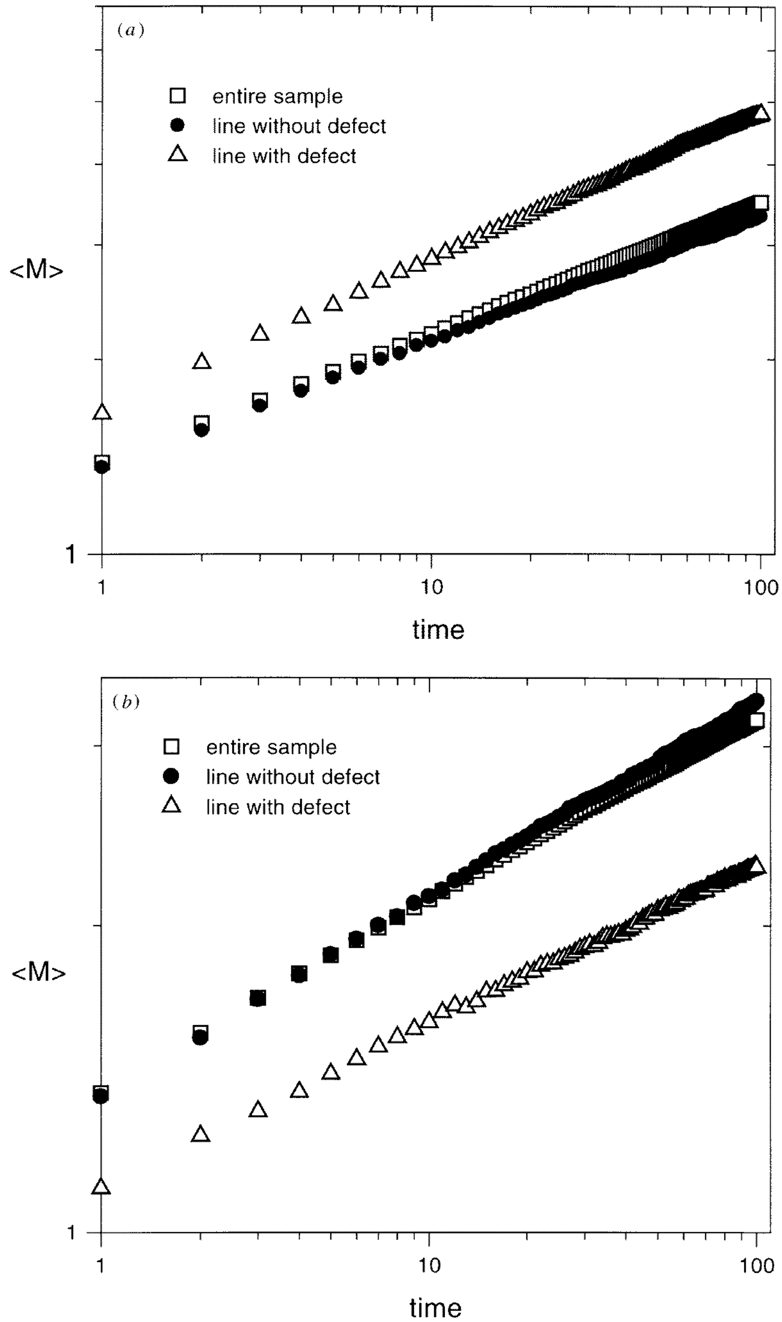

Figure 1. Log–log plot of the magnetization versus time for: ● a pure line, △ the line of defects, and □ the complete square lattice. Samples (100 000) were originally at high temperature but they had a small magnetization per site $m_0 = 0.0278$. The size of the lattice in this case is $L = 72$ and the updating was done with heat-bath dynamics. (*a*) Results for the case $(J'/J) = 2$ whereas (*b*) presents results for $(J'/J) = 0.5$.

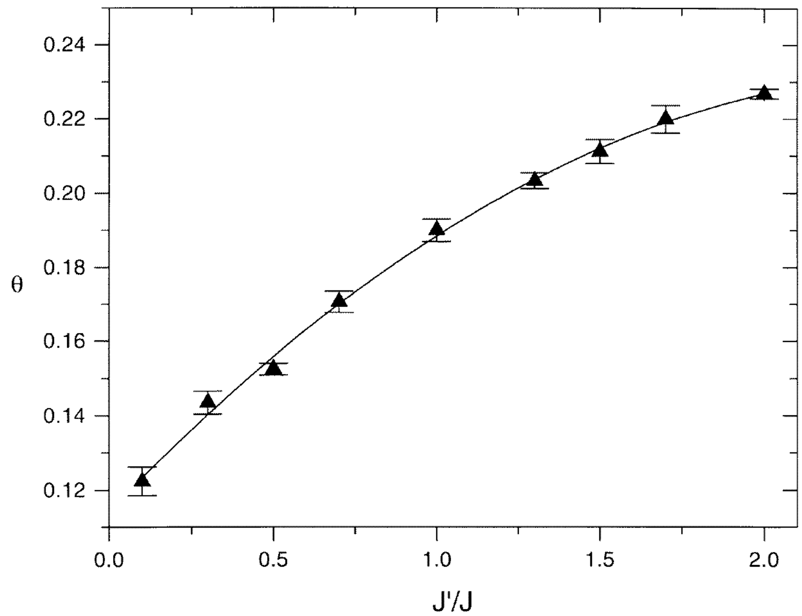

Figure 2. Dependence of the new critical exponent θ on the ratio of couplings (J'/J). The full curve is a guide for the eyes.

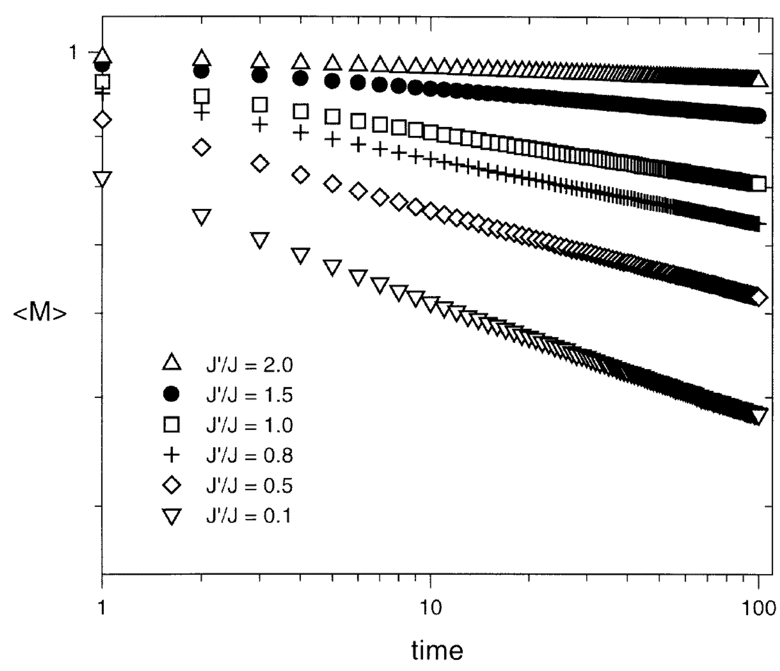

Figure 3. Log–log plot of temporal decay of the magnetization at the line of defects for several values of the ratio of couplings (J'/J). The slope of those curves depends on the ratio (J'/J) and give estimates for η^* (equation (4)). Heat-bath dynamics was used to update samples (50 000) which were completely magnetized at $t = 0$. The size of lattice was $L = 36$.

the magnetization of samples that at $t = 0$ are totally magnetized ($m = 1$), is given by

$$m \sim t^{-\eta/2\nu z} \tag{3}$$

where z is the dynamical critical exponent, and η and ν are the known static indices of the model. If our hypothesis is correct we can obtain the exponent η^* which characterizes the polynomial decay of the correlation function (at $T = T_c$) along the line of defects by comparing the slope of two straight lines in the log–log plot of magnetization versus time (one of them belonging to the line of defects, the other corresponding to a normal line far enough off the altered one). Figure 3 shows the behaviour of the magnetization of those lines, for several values of the coupling constant J'. Figure 4 presents estimates for the exponent η^*, obtained by the relation

$$\eta^* = \eta \frac{\tan \alpha'}{\tan \alpha} \tag{4}$$

where $\eta = \frac{1}{4}$ is the critical exponent for the pure Ising model. To obtain that equation we have used the fact that z always has the same value, despite the value of J'. In addition, $\tan \alpha'$ and $\tan \alpha$ are the slopes of the curves for altered and pure lines. The results so obtained are in complete agreement with the exact ones (full curve) obtained by Bariev [10] and McCoy and Perk [11]. To obtain the error bars (smaller than the size of points) we repeat each simulation five times. We observe that our estimates are independent of the specific value of the dynamical critical exponent z.

Another way to obtain the same conclusion is to calculate the evolution of the second moment of the magnetization of samples which at $t = 0$ satisfy the conditions: $m = 0$ and $\xi = 0$. Scaling arguments [3] assert that m^2 should behave as

$$\langle m^2 \rangle \sim t^{(d-\eta/\nu)/z} \tag{5}$$

where d is the dimension of the object for which we are calculating that quantity ($d = 1$ when we are interested in lines). Figure 5 illustrates the results for several values of J'/J, this figure shows polynomial behaviour with distinct exponents. Finally, we can estimate η^* from the equation

$$\eta^* = 1 - (1 - \eta) \frac{\tan \phi'}{\tan \phi} \tag{6}$$

where $\tan \phi'$ ($\tan \phi$) are the slopes of the straight lines in the figure, corresponding to the line with (without) defects. Results for η^* obtained by this procedure are plotted in figure 6. Once again we see that the hypothesis of constant z works well, at least when $J'/J < 1$. We attribute the disagreement observed for higher values of the ratio J'/J to the influence of the strong coupling over the fluctuations of the magnetization of the pure line.

Finally, we check the behaviour of the anomalous dimension x_0 of the magnetization far from equilibrium. As it is now known, the critical initial slip is a consequence of the difference between the anomalous dimension x_0 and the usual scaling dimension $\eta/2\nu$ of the magnetization [22]. More specifically, we have

$$x_0 = \eta/2\nu + z\theta \tag{7}$$

which permits us to obtain x_0, for any value of the ratio (J'/J), since we know both: the exponent η (figure 4) and the new dynamical exponent θ (figure 2). As shown in figure 7, the result is very approximately equal to 0.53 (corresponding to the pure case) for a large interval of (J'/J). We have used the value 2.172 ± 0.006, obtained by Grassberger [23], for the dynamical critical exponent z.

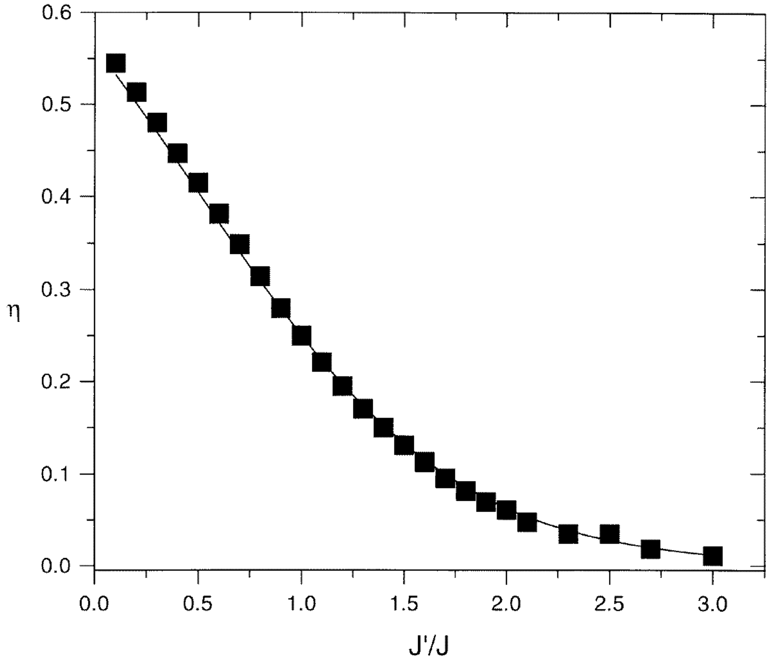

Figure 4. Estimates for the nonuniversal critical exponent η^* (characterizing the decay of the correlation function at the defect line) obtained by the scaling relation $m \sim t^{-\eta/2\nu z}$, exhibited in figure 3 and equation (4). The full curve is the exact result for η^*, given by equation (2).

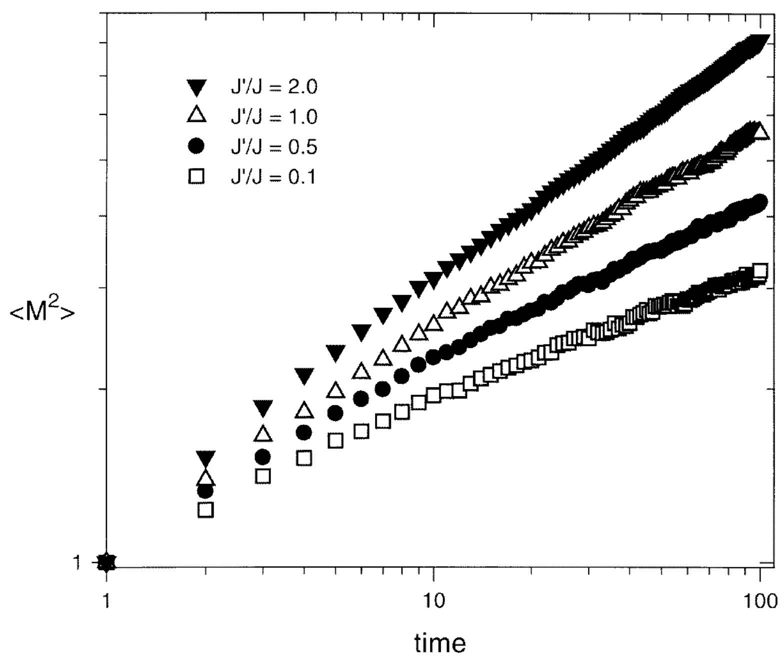

Figure 5. Log–log plot of the squared magnetization at the defect line for several values of the ratio of couplings (J'/J). Scaling relations predict that the slope of the lines should be given by $(d - \eta^*/\nu)/z$.

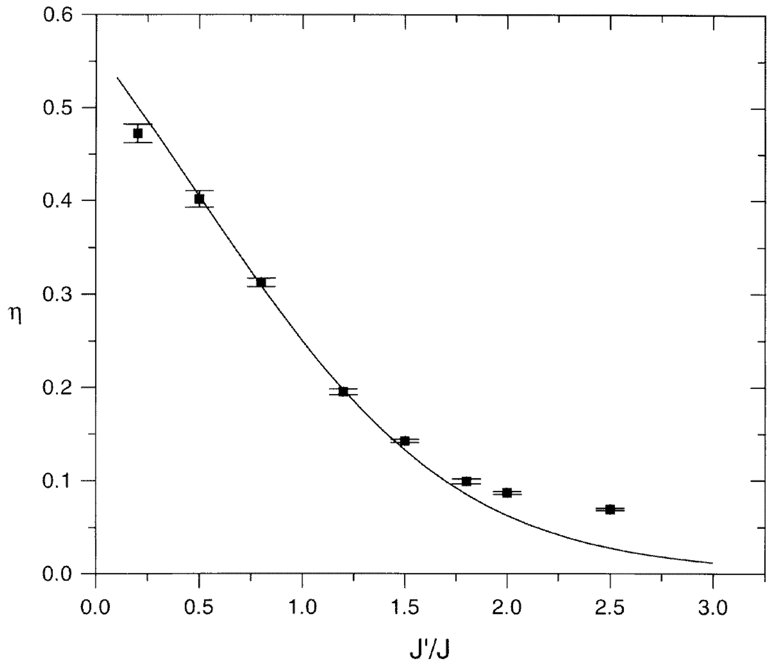

Figure 6. Estimates for the nonuniversal critical exponent η^* obtained with equation (6). When J'/J is greater than 1 results are poor.

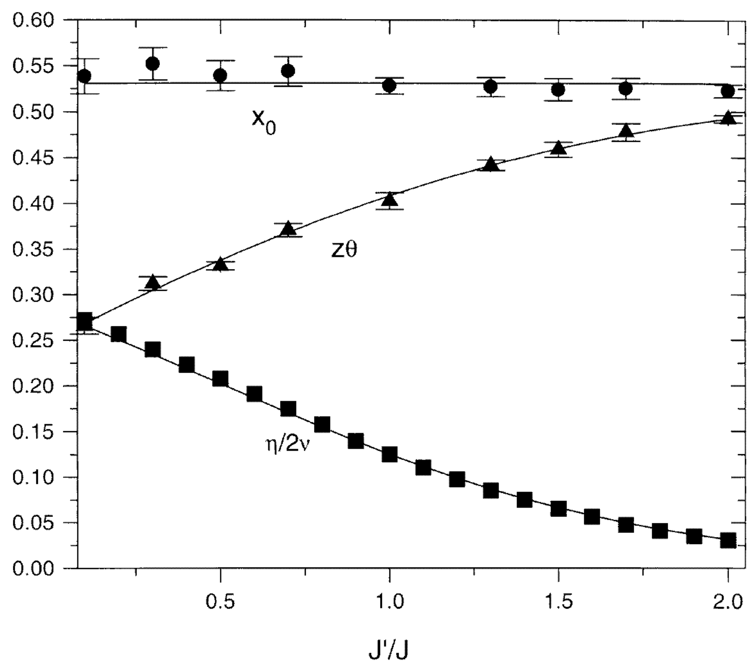

Figure 7. Plot of the anomalous dimension of the magnetization $x_0 = z\theta + \eta^*/2\nu$ versus the ratio of couplings (J'/J).

In summary, in order to understand the effect of a marginal operator over the dynamical critical exponents we have investigated the short-time critical dynamics of the Ising model with a line of defects. We have confirmed nonuniversality for the 'local' exponent θ but not for the global dynamical exponent z. The universality of z permitted us to determine the static nonuniversal index η^*, of the correlation function along the line of defects, using just the first 100 steps of the simulation (free of critical slowing down phenomena). We have also detected an apparent universal behaviour for the anomalous dimension x_0 of the magnetization, even at the line of defects. Although the numerical results can be slightly different, they are qualitatively the same when we use heat-bath or Metropolis updating. At present, we are studying global and local persistence [24–26] phenomena in this model.

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